Std : 12th **Mathematics and Statistics** Time : 3 Hrs.

SECTION- A

Q.1) Select and write the most appropriate answer from the given alternatives for each sub-question: [16]

i) $\int[\sin(\log x)+\cos(\log x)]dx =$

(A) x cos (log x) + c (B)sin (lo x) + c (C) cos (log x) + c (D) x sin (log x) + c

ii) The area enclosed between the curve $y = \cos 3x, 0 \le x \le \frac{\pi}{6}$ and the X-axis is

(A) $\frac{1}{2}$ sq. units (B) 1 sq. units (C) $\frac{2}{3}$ sq. units (D) $\frac{1}{3}$ sq. units

iii) The differential equation of $y = c^2 + \frac{c}{x}$ = ----------

(A) $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = y$ (B) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(C) $x^3\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = y$ (D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

iv) If p.m. f. of a d.r. v. X is

P(X = x) = $\frac{x}{n(n+1)}$, for x = 1, 2, 3,----- n and

= 0, otherwise

then E(X) =

(A) $\frac{n}{1} + \frac{1}{2}$ (B) $\frac{n}{3} + \frac{1}{6}$ (C) $\frac{n}{2} + \frac{1}{5}$ (D) $\frac{n}{1} + \frac{1}{3}$

v) If $A = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, adj $A = \begin{bmatrix}4 & a\\ -3 & b\end{bmatrix}$, then the values of a and b are

(A) a = -2, b = 1 (B) a = 2, b = 4 (C)a = 2, b = -1 (D) a = 1, b = -2

vi) If $\cos\alpha, \cos\beta, \cos\gamma$ are the directions cosines of a line, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

(A) 1 (B) 2 (C) 3 (D) 4

vii) The negation of $p \wedge (q \to r)$ is ----------

(A) $\sim p \wedge (\sim q \to r)$ (B) $p \vee (\sim q \vee r)$

(C) $\sim p \wedge (\sim q \to \sim r)$ (D) $\sim p \vee (\sim q \wedge \sim r)$

viii) The angle between the planes $\bar{r}.(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + 4 = 0$ and $\bar{r}.(2\hat{\imath} + \hat{\jmath} - 3\hat{k}) + 7 = 0$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\cos^{-1}\left(\frac{3}{4}\right)$ (D) $\cos^{-1}\left(\frac{9}{14}\right)$

Q.2) Answer the following: [4]

i) Find the distance of the point (4, 3, 5) from the Y-axis.

ii) Find the value of p if the equation $px^2-8xy+3y^2+14x+2y-8=0$ represents a pair of perpendicular lines.

iii) If $s = 60 + 2t - 10t^2$ is the displacement of the particle at time t, then find the rate of change in displacement w.r.t. t.

iv) The following is the c.d.f. of a discrete r.v. X.

X	-3	-1	0	1	3	5	7	9
F(x)	0.1	0.3	0.5	0.65	0.75	0.85	0.90	1

Find P(X=0).

SECTION B

Attempt any EIGHT of the following questions: [16]

3) Using truth table, varify that $\sim(P \vee q) \equiv \sim p \wedge \sim q$

4) Find the matrix X such that AX = B, where $A = \begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$ and $B = \begin{bmatrix}0 & 1\\ 2 & 4\end{bmatrix}$

5) Find the cartesian co-ordinates of the point whose polar co-ordinates are $\left(\frac{3}{4}, \frac{3\pi}{4}\right)$

6) If $|\bar{a}| = |\bar{b}| = 1, \bar{a}.\bar{b} = 0$ and $\bar{a} + \bar{b} + \bar{c} = 0$, then find $|\bar{c}|$.

7) If $\bar{c} = 3\bar{a} - 2\bar{b}$, then prove that $[\bar{a}\ \bar{b}\ \bar{c}] = 0$

8) Find the acute angle between lines $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2}$ and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}$

9) Differentiate the following w.r.t. x : $\cos^{-1}(4\cos^3 x - 3\cos x)$

10) Evaluate : $\int \sqrt{1-\cos 2x}\ dx$

11) Evaluate : $\int \frac{1}{1+x-x^2}\ dx$

12) Evaluate : $\int_{-3}^{3} \frac{x^3}{9-x^2}\ dx$

13) Find the area of the region bounded by the following curve, X -axis and the given lines: y2 = 16x, x = 0 , x = 4

14) Solve the following differential equation. $\log\left(\frac{dy}{dx}\right) = 2x + 3y$

SECTION C

Attempt any EIGHT of the following questions: **[24]**

15) Construct the switching circuit of the following : $(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$

16) Find the general solution of $\tan^3\theta = 3\tan\theta$

17) Find the vector projection of $\overline{PQ}$ on $\overline{AB}$ where P, Q, A, B are the points (-2, 1, 3), (3, 2, 5), (4, -3, 5) and (7, -5, -1) respectively.

18) Show that vector area of a quadrilateral ABCD is $\frac{1}{2}(\overline{AC} \times \overline{BD})$, where AC and BD are its diagonals.

19) If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, then find k.

20) Findthe cartesian equation of the plane $\bar{r} = (5\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$

21) If $log_{10}\left(\frac{x^3-y^3}{x^3+y^3}\right) = 2$, show that $\frac{dy}{dx} = \frac{-99x^2}{101y^2}$

22) Find $\frac{dy}{dx}$, if $y = \tan^{-1}(\text{cosec } x + \cot x)$

23) The surface area of a spherical balloon is increasing at the rate of 2 cm^2 / sec. At what reate the volume of the balloon is increasing when radius of the balloon is 6 cm ?

24) Solve : $\frac{dy}{dx} = \cos(x + y)$

25) Solve the differential equation: $\frac{dy}{dx} + y = e^{-x}$

26) For the following p.d.f. of r.v. X, find :

i) P(X< 1) ii) P(|X | < 1)

if f(x) = $\frac{x^2}{18}$, foe -3 < x < 3

= 0, otherwise

SECTION D

Attempt any FIVE of the following questions: **[20]**

27) Find the inverse of $A = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ by elementary row transformations.

28) In ΔABC, prove that $\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$

29) Show that a homogeneous equation of degree 2 in x and y i.e. $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines through the origin if $h^2 - ab \geq 0$.

30) Maximize: Z = 3x + 5y subject to $x + 4y \leq 24$, $3x + y \leq 21$, $x + y \leq 9$, $x \geq 0$, $y \geq 0$. Also find maximum value of Z.

31) Verify Lagrange's mean value theorem for the following functions.

f(x) = (x -1) (x - 2) (x - 3) on [0, 4]

32) Prove that : $\int \sqrt{a^2 - x^2}\ dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

33) In binomial distribution with five Bernoulli's trials, probability of one and two success are 0.4096 and 0.2048 respectively. Find probability of success.

34) Show that : $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$, if f(x) is an even function

= 0, if f(x) is an odd function