Smart 17A | Marks 80

Std : 12th Mathematics and Statistics Time : 3 Hrs.

SECTION- A

Q.1) Select and write the most appropriate answer from the given alternatives for each sub-question: [16]

i) If the line $\frac{x+1}{2} = \frac{y-m}{3} = \frac{z-4}{6}$ lies in the plane 3x-14y+6z+49=0, then the value of m is

(A) 5 (B) 3 (C) 2 (D) -5

ii) If y = sec ($\tan^{-1}$ x) then $\frac{dy}{dx}$ at x = 1, is equal to

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

iii) $\int \frac{1}{x+x^5} dx = f(x) + c$, then $\int \frac{x^4}{x+x^5} dx =$

log | x | - f(x) + c (B) f(x) + log | x | + c (C) f(x) - log | x | + c (D) $\frac{1}{5} x^5 f(x) + c$

iv) The integrating factor of linear differential equation $x \frac{dy}{dx} + 2y = x^2 \log x$ is --------

(A) $\frac{1}{x}$ (B) k (C) $\frac{1}{n^2}$ (D) x^2

v) If X-B(4, p) and P(X = 0) = $\frac{16}{81}$, then P(X = 4) = ----------

(A) $\frac{1}{16}$ (B) $\frac{1}{81}$ (C) $\frac{1}{27}$ (D) $\frac{1}{8}$

vi) In ΔABC, if $c^2 + a^2 - b^2 = ac$, then ∠B= ----------

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

vii) If $(p \wedge \sim q) \rightarrow (\sim p \vee r)$ is a false statement, then the respective truth values of p, q and r are

(A) T, F, F (B) F, T, T (C) T, T, T (D) F, F, F

viii) If θ be the angle between any two vectors $\bar{a}$ and $\bar{b}$, then $|\bar{a} . \bar{b}| = |\bar{a} \times \bar{b}|$, when θ is equal to

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

Q.2) Answer the following: [4]

i) If A = {1, 2, 3, 4, 5, 6 , 7, 8, 9}, determine the truth value of the following statement:
$\exists x \in$ A such that x + 8 = 15.

ii) Find the direction cosines of any normal to the XY plane.

iii) If f(2) = 27, f ' (2) = 40 and h = 0.01, then find the approximate value of f(2.01).

iv) For the following probability distribution, find k.

X = x	1	2	3
P(X = x)	K	2k	3k

SECTION B

Attempt any EIGHT of the following questions: [16]

3) Find the general solution of cosec θ= $-\sqrt{2}$

4) Find the adjoint of the matrix $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$

5) Find k, if the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product.

6) Find $\bar{a}.(\bar{b} \times \bar{c})$, if $\bar{a} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}, \bar{b} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\bar{c} = -5\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

7) Find the Cartesian equation of the line passing through points having position vectors $3\hat{\imath} + 4\hat{\jmath} - 7\hat{k}$ and $6\hat{\imath} - \hat{\jmath} + \hat{k}$.

8) Find the Cartesian equation of the plane passing through A (7, 8, 6) and parallel to the plane $\bar{r}.(6\hat{\imath} + 8\hat{\jmath} + 7\hat{k}) = 0$.

9) Find $\frac{dy}{dx}$, if $xe^y + ye^x = 1$

10) Test whether the following function is increasing or decreasing. $f(x) = x^3 - 6x^2 + 12x - 16$, $x \in R$

11) Evaluate: $\int \frac{\tan x}{\sec x + \tan x} dx$

12) Differentiate the following w.r.t. x: $\cot^3 [\log (x^3)]$

13) Evaluate : $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

14) Solve the differential equation. $\sec^2 x.\tan y.dx + \sec^2 y.\tan x.dy = 0$

SECTION C

Attempt any EIGHT of the following questions: **[24]**

15) Examine whether the following statement pattern is a tautology or a contradiction or a contingency : $(\sim p \rightarrow q) \wedge (p \wedge r)$

16) Find the general solution of $\sec^2 2\theta = 1 - \tan 2\theta$

17) With usual notations prove that $2\left\{a \sin^2 \frac{C}{2} + c \sin^2 \frac{A}{2}\right\}$ = a - b + c.

18) If Q is the foot of the perpendicular from P(2, 4, 3) on the line joining the points A(1, 2, 4) and B(3, 4, 5), find coordinates of Q.

19) Prove that $\left[\bar{a} \;\; \bar{b} + \bar{c} \;\; \bar{a} + \bar{b} + \bar{c}\right] = 0$

20) Find the vector equation of the plane which makes intercepts 1, 1, 1 on the co-ordinates axes.

21) If y = f(u) is a differentiable function of u and u = g(x) is a differentiable function of x, then prove that the composite function y = f[g(x)] is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

22) Find the value of x for which the function $f(x) = x^3 - 12x^2 - 144x + 13$ is increasing.

23) If each side of an equilateral triangle increases at the rate of $\sqrt{2}$ cm/ sec, find the rate of increase of its area when its side is of length 3 cm.

24) Obtain the differential equation by eliminating arbitrary constant A and B.
y = A cos (log x) + B sin (log x)

25) Solve : $x + y\frac{dy}{dx} = \sec(x^2 + y^2)$

26) A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. Find the probability that he will win a prize at least once.

SECTION D

Attempt any FIVE of the following questions: **[20]**

27) Solve the following equations by the method of reduction:
x + 3y + 2z = 6, 3x - 2y + 5z =5, and 2x - 3y + 6z = 7

28) Prove that three vectors $\bar{a}$, $\bar{b}$ *and* $\bar{c}$ are coplanar, if and only if there exists a non-zero linear combination $x\bar{a} + y\bar{b} + z\bar{c} = 0$ with (x, y, z) ≠ (0, 0, 0).

29) Show that lines $\bar{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ *and* $\bar{r} = (4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ are coplanar, Find the equation of the plane determined by them.

30) A company manufactures bicycles and tricycles, each of which must be processed through two machines A and B. Maximum availability of Machine A and B is respectively 120 and 180 hours. Manufacturing a bicycle requires 6 hours on Machine A and 3 hours on Machine B. Manufacturing a tricycle requires 4 hours on Machine A and 10 hours on Machine B. If profits are Rs.180/- for a bicycle and Rs.220/- for a tricycle. Determine the number of bicycle and tricycles that should be manufactured in order to maximize the profit.

31) If u and v are two functions of x, then prove that $\int u.vdx = u\int vdx - \int \left(\frac{d}{dx}(u).\int v\,dx\right)dx$
Hence, Evaluate $\int x \log x\, dx$

32) Find the area of the region lying between the parabolas $4y^2 = 9x$ and $3x^2 = 16y$.

33) Find the expected value, variance and standard deviation of r.v. X for the following p.m.f.

X = x	0	1	2	3	4	5
P(X=x)	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

34) Find the equation of the curve which passes through the origin and has the slope x + 3y - 1 at any point (x, y) on it.

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