Std. : **XII (Sci.)**

First Preliminary Exam - 2022

Sub. : **Mathematics**

Time : **3.00** Hrs.

Marks : **80**

Instructions :

The question paper is divided into **four** sections A, B, C, D.

(1) **Section A :** Q. No. 1 contains eight multiple choice type questions carrying **two** marks each. Q. No. 2 contains **four** very short answer type of questions carrying **one** mark each.

(2) **Section B :** Contains **twelve** short answer type questions carrying **two** marks each. (Attent any **eight**)

(3) **Section C :** Contains **twelve** short answer type questions carrying **three** marks each. (Attent any **eight**)

(4) **Section D :** Contains **eight** long answer type questions carrying **four** marks each. (Attent any **five**)

(5) Use of logarithmic table is allowed. Use of calculator is not allowed.

(6) Figures to the right indicate full marks.

(7) Use of graph paper is not necessary. Only rough sketch is expected.

(8) For each MCQ. correct answer must be written along with its alphabet : e.g. (a) ..../(b) ..../(c) ..../(d) ..... Only first attempt will be considered for evaluation.

(9) Start answer to each section on new page.

Section A

Q.1 Select and write the most appropriate answer from the given alternatives. **(16)**

(i) Statement $P \rightarrow (q \rightarrow P)$ is equivalent to

(a) $P \rightarrow (P \rightarrow q)$ (b) $P \rightarrow (P \vee q)$

(c) $P \rightarrow (P \wedge q)$ (d) $P \rightarrow (P \leftrightarrow q)$

(ii) In ΔABC, if $\text{Cos } A = \frac{\text{Sin } B}{2 \text{ Sin } C}$ then ΔABC is .....

(a) an equilateral triangle (b) right angled triangle

(c) an isoscles triangle

(d) an isosceles right angled triangle

(d) .....

(iii) If $|\bar{a}| = 3$, $|\bar{b}| = 5$, $|\bar{a}+\bar{b}| = 4$ then $|\bar{a}-\bar{b}|$ is equal to

(a) 2 (b) $4\sqrt{3}$ (c) $2\sqrt{13}$ (d) $\sqrt{13}$

(iv) $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)^3}\, dx =$

(a) $\frac{1}{8}$ (b) $\frac{-1}{8}$ (c) $\frac{3}{8}$ (d) $\frac{5}{8}$

(v) The sum of slopes of lines given by $x^2 - 2\lambda xy - 7y^2 = 0$ is 4 times their product, then value of λ is

(a) 2 (b) −1 (c) 1 (d) −2

(vi) The angle between the lines

$\bar{r}(\bar{i}+2\bar{j}-3\bar{k}) + \lambda(3\bar{i}+2\bar{j}+6\bar{k})$ and

$r(5\bar{i}-2\bar{j}+7\bar{k}) + \mu(\bar{i}+2\bar{j}+2\bar{k})$ is

(a) $\cos^{-1}\left(\frac{17}{21}\right)$ (b) $\cos^{-1}\left(\frac{20}{21}\right)$

(c) $\cos^{-1}\left(\frac{18}{21}\right)$ (d) $\cos^{-1}\left(\frac{19}{21}\right)$

(vii) If $y = \cot^{-1}\left(\frac{1+6x^2}{x}\right)$ then $\frac{dy}{dx} =$

(a) $\frac{1}{1+9x^2} - \frac{1}{1+4x^2}$ (b) $\frac{3}{1+9x^2} - \frac{2}{1+4x^2}$

(c) $\frac{6}{1+9x^2}$ (d) $\frac{12}{1+9x^2}$

(viii) If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{x}{4}$ lies in the plane $4x + 4y - kz = 0$ then value of k is

(a) 4 (b) 5 (c) 6 (d) 7

Q. 2 Answer the following. **(4)**

(i) If a = 4, b = 5 and $\sin A = \frac{4}{5}$, find m ∠B.

(ii) Write contra positive of the statement $(p \vee q) \rightarrow r$

(iii) A line makes angles α, β, γ with positive direction of co-ordinate axes then find value of

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

(iv) Find order and degree of the differential equation

$$\sqrt{1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$$

Section B

Attempt any EIGHT of the following. **(16)**

Q. 3 If θ is acute angle between the lines $3x^2 - 4xy + by^2 = 0$ and $\tan\theta = \frac{1}{2}$, find b.

Q. 4 Find volume of tetrahedron whose coterminous edges are $4\bar{i} - 4\bar{j} - 2\bar{k}$, $3\bar{i} - \bar{j}$ and $-4\bar{j} + \bar{k}$

Q. 5 Solve the differntial equation $x + y\frac{dy}{dx} = x^2 + y^2$

Q. 6 Evaluate : $\int \frac{(x+1)\,e^x}{\cos^2(xe^x)}\,dx$

Q. 7 Find the direction ratios of a line perpendicular to both the lines whose direction ratios are 3, 2, −1 and 2, 4, −2

Q. 8 Find cartesion co-ordinates of the point whose polar co-ordinates are $\left(\frac{3}{4}, 135^\circ\right)$

Q. 9 Find value of a if $\int_2^a (x+1)\,dx = \frac{7}{2}$

Q. 10 Find the inverse by adjoint method $A = \begin{bmatrix} -1 & 5 \\ -3 & 8 \end{bmatrix}$

Q. 11 Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Q. 12 If displacement of a partical moving along a straight line is given by $x = 3t^4 - 20t^3 + 36t^2 + 5$ Find the time when particle stops.

Q. 13 The probability of a random variable X is given by

$X = x$	-1	2	a
$P(X = x)$	$\frac{1}{2}$	$\frac{3}{8}$	b

and $E(X) = \frac{5}{8}$, find values of a and b.

Q. 14 A fait coine is tossed 6 times. Find the probability of getting 2 heads.

Section C

Attempt any EIGHT of the following. **(24)**

Q. 15 Show that the points A (2, 1, −1), B (0, −1, 0), C (4, 0, 4) and D (2, 0, 1) are coplanar.

Q. 16 At what point does the curve $y = 3x - x^2$ have slope −5 ?

Q. 17 If θ is measure of acute angle between the pair of lines $ax^2 + 2hxy + by^2 = 0$ than prove that

$$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|, \quad a + b \neq 0$$

Q. 18 Find the vector equation of line passing through the point $\bar{i} + 2\bar{j} + 3\bar{k}$ and perpendicular to the vectors $\bar{i} + \bar{j} + \bar{k}$ and $2\bar{i} - \bar{j} + \bar{k}$

Q. 19 Evaluate : $\int \frac{1}{3 + 5\cos x}\, dx$

Q. 20 Solve the differential equation

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$$

Q. 21 Prove that :

$$\int_0^{2a} f(x)\, dx = \int_0^{a} f(x)\, dx + \int_0^{a} f(2a - x)\, dx$$

Q. 22 The probability distribution of r.v X is given by

$X = x$	0	1	2	3	4
$P(X = x)$	0.45	0.35	0.15	0.03	0.02

Find variance of X.

Q. 23 Evaluate : $\int (\log x)^2 \, dx$

Q. 24 Find the joint equation of pair of lines passing through origin and perpendicular to the lines

$5x^2 + 2xy - 3y^2 = 0$

Q. 25 If sum of mean and variance of a binomial distribution is $\frac{25}{9}$ for 5 trials, find P.

Q. 26 If $y = \text{Sin}^{-1} x$, then show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

Section D

Attempt any FIVE of following. **(20)**

Q. 27 Solve the following equations by inversion method.

$x + y + z = -1$, $x - y + z = 2$, $x + y - z = 3$

Q. 28 Using vector method prove that perpendicular bisector of sides of a triangle are concurrent.

Q. 29 min $Z = 8x + 10y$

Subject to $2x + y \geq 7$, $2x + 3y \geq 15$,

$y \geq 2$, $x \geq 0$, $y \geq 0$, solve graphically

Q. 30 Find the area enclosed by parabola $y^2 = x$ and the line $x + y = 2$.

Q. 31 If $x = f(t)$, $y = g(t)$ are differentiable functions of t, then prove that

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad , \quad \frac{dx}{dt} \neq 0$$

Hence find $\frac{dy}{dx}$ if $x = a\cos^2 t$, $y = a\sin^2 t$

Q. 32 A rod of 108 m long is bent to form a rectangle. Find it's dimensions if it's area is maximum.

Q. 33 Find the general solution of

$$\text{Cos } \varkappa - \text{Sin } \varkappa = 1.$$

Q. 34 Examine whether the statement $(\sim p \rightarrow q) \wedge (p \wedge r)$ is tautology, contradiction or contigency.

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