



Instructions: The question paper is divided into four sections -

- (1) Section A: Q. no. 1 contains eight multiple choice type questions carrying two marks each.  
Q. no. 2 contains four very short answer type of questions carrying one mark each.
- (2) Section B: Contains twelve short answer type questions carrying two marks each. (Attempt any eight)
- (3) Section C: Contains twelve short answer type questions carrying three marks each. (Attempt any eight)
- (4) Section D: Contains eight long answer type questions carrying four marks each. (Attempt any five)
- (5) Use of logarithmic table is allowed. Use of calculator is not allowed.
- (6) Figures to the right indicate full marks.
- (7) Use of graph paper is not necessary. Only rough sketch is expected.
- (8) For each MCQ, correct answer must be written along with its alphabet :  
e.g. (a) / (b) / (c) / (d). Only first attempt will be considered for evaluation.
- (9) Start answer to each section on new page.

#### SECTION - A

Q.1. Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: (16)

- (i) If  $p \rightarrow (-p \vee q)$  is false, then the truth values of  $p$  and  $q$  respectively are:  
(a) F, T (b) F, F (c) T, T (d) T, F
- (ii) The general solution of  $\cot 4x = -1$  is  
(a)  $x = n\pi + \frac{3\pi}{4}, n \in \mathbb{Z}$  (b)  $x = \frac{n\pi}{4} + \frac{3\pi}{16}, n \in \mathbb{Z}$  (c)  $x = \frac{n\pi}{4} + \frac{\pi}{16}, n \in \mathbb{Z}$  (d)  $x = \frac{n\pi}{8} + \frac{3\pi}{16}, n \in \mathbb{Z}$
- (iii) If  $|a| = 3, |b| = 5$  and  $|a + b| = 4$  then  $|a - b|$  is equal to:  
(a) 2 (b)  $4\sqrt{13}$  (c)  $2\sqrt{13}$  (d)  $\sqrt{13}$
- (iv)  $\int_0^1 \frac{x^2}{1+x^2} dx =$   
(a)  $\frac{\pi}{4} - 1$  (b)  $\frac{\pi}{2} - 1$  (c)  $1 - \frac{\pi}{2}$  (d)  $1 - \frac{\pi}{4}$
- (v) The sum of the slopes of the lines given by  $x^2 - 2\lambda xy - 7y^2 = 0$  is 4 times their product, then the value of  $\lambda$  is:  
(a) 2 (b) -1 (c) 1 (d) -2
- (vi) The angle between the lines  $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$  and  $\vec{r} = (5\hat{i} - 2\hat{j} + 7\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$  is  
(a)  $\cos^{-1}\left(\frac{17}{21}\right)$  (b)  $\cos^{-1}\left(\frac{20}{21}\right)$  (c)  $\cos^{-1}\left(\frac{18}{21}\right)$  (d)  $\cos^{-1}\left(\frac{19}{21}\right)$
- (vii) If  $y = \tan^{-1}\left(\frac{6x-7}{6+7x}\right)$ , then  $\frac{dy}{dx} =$   
(a)  $\frac{6}{1+(6+7x)^2}$  (b)  $\frac{7}{1+(6+7x)^2}$  (c)  $\frac{1}{1+x^2}$  (d)  $\frac{6}{1+x^2}$
- (viii) If the line  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$  lies in the plane  $4x + 4y - Kz = 0$ , the value of  $K$  is.  
(a) 4 (b) 5 (c) 6 (d) 7

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Q.2. Answer the following:

- (i) Find the principal solution of the equation  $\sec x = \frac{2}{\sqrt{3}}$ .
- (ii) Write the negation of  $(p \leftrightarrow q)$ .
- (iii) A line makes angles  $\alpha, \beta, \gamma$  with the positive directions of coordinate axes, then find the value of  $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ .
- (iv) Find the order and degree of the differential equation  $\frac{d^2 y}{dx^2} = \sqrt{1 - \left(\frac{dy}{dx}\right)^4}$ .

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### SECTION - B

Attempt any EIGHT of the following :

- Q.3. If  $\theta$  is the acute angle between the lines given by  $3x^2 - 4xy + by^2 = 0$  and  $\tan \theta = \frac{1}{2}$ , find  $b$ . (16)
- Q.4. Find the volume of the parallelepiped whose coterminal edges are  $2\hat{j} - 3\hat{j} + \hat{j} - \hat{k}$  and  $3\hat{j} - \hat{k}$ .
- Q.5. Find the general solution of the differential equation  $\tan y \cdot \frac{dy}{dx} = \sin(x+y) - \sin(x-y)$ .
- Q.6. Evaluate  $\int \sin^3 x \cos^3 x dx$ .
- Q.7. Find the direction ratios of a line perpendicular to both the lines whose direction ratios are  $3, 2, -1$  and  $2, 4, -2$ .
- Q.8. Solve the equation :  $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2} \tan^{-1} x$ , for  $x > 0$ .
- Q.9. Find the value of 'a' if  $\int_1^2 (x+1) dx = \frac{7}{2}$ .
- Q.10. Find the inverse of the matrix  $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$  by adjoint method.
- Q.11. Differentiate  $e^x \cos x$  w.r.t.  $e^{-x} \sin x$ .
- Q.12. The side of a square is increasing at the rate of 0.5 cm/sec. find the rate of increase of the perimeter when the side of the square is 10 cm. long.
- Q.13. The p.m.f. of random variable  $X$  is as follows.  
 $P(X=0) = 5k^2, P(X=1) = 1-4k, P(X=2) = 1-2k$  and  $P(X=x) = 0$  for any other value of  $X$ . Find  $k$ .
- Q.14. A fair coin is tossed 6 times. Find the probability of getting heads 4 times.

### SECTION - C

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Attempt any EIGHT of the following :

- Q.15. Show that the points  $A(2,1,-1)$ ,  $B(0,-1,0)$ ,  $C(5,0,4)$  and  $D(2,0,1)$  are coplanar.
- Q.16. Find the value of  $x$  such that  $f(x) = 2x^3 - 15x^2 - 84x - 7$  is a decreasing function.
- Q.17. Find the joint equation of the pair of lines through the origin and perpendicular to the lines given by  $2x^2 + 7xy + 3y^2 = 0$ .
- Q.18. Find the vector equation of a line passing through the point  $\hat{i} + 2\hat{j} + 3\hat{k}$  and perpendicular to the vectors  $\hat{i} + \hat{j} + \hat{k}$  and  $2\hat{j} - \hat{j} + \hat{k}$ .
- Q.19. Evaluate :  $\int (\log x)^2 dx$ .
- Q.20. Solve the differential equation  $1 + \frac{dy}{dx} = \operatorname{cosec}(x+y)$ .
- Q.21. Evaluate :  $\int_0^{\pi/2} \frac{\sin x}{(1+\cos x)^2} dx$ .
- Q.22. Given below is the probability distribution of a discrete random variable  $x$ .
- |          |     |   |      |      |     |      |
|----------|-----|---|------|------|-----|------|
| $X$      | 1   | 2 | 3    | 4    | 5   | 6    |
| $P(X=x)$ | $k$ | 0 | $2k$ | $5k$ | $k$ | $3k$ |
- Q.23. If the sum of mean and variance of a binomial distribution is  $\frac{25}{9}$  for 5 trials, find  $p$ .
- Q.24. Evaluate :  $\int \frac{1}{4-5\cos x} dx$ .
- Q.25. If the acute angle between the lines  $x^2 - 2hxy + y^2 = 0$  is  $60^\circ$ , find  $h$ .
- Q.26. If  $y = \sin^{-1} x$  show that  $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = 0$ .

### SECTION - D

Attempt any FIVE of the following :

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- Q.27. Simplify and show that the statement:  $[p \wedge (\sim p \vee q)] \vee (\sim p \wedge q) \vee [(p \vee \sim q) \wedge r]$  is equivalent to  $(q \vee r)$ .
- Q.28. Solve the following equations by the method of inversion.  $x+y+z = -1, x-y+z = 2, x+y-z = 3$ .
- Q.29. Find the general value of  $\theta$ . If  $\cot \frac{\theta}{2} = \operatorname{cosec} \frac{\theta}{2} = \cot \theta$ .
- Q.30. Using vector method prove that the perpendicular bisectors of the sides of a triangle are concurrent.
- Q.31. Minimize  $z = 8x + 10y$ , subject to  $2x + y \geq 7, 2x + 3y \geq 15, y \geq 2, x \geq 0, y \geq 0$ .
- Q.32. If  $\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \tan^{-1} a$ . Show that  $\frac{dy}{dx} = \frac{y}{x}$ .
- Q.33. A rectangular sheet of paper has the area 24 square meters. The margin at the top and bottom is 75 cm and sides 50 cm each. What are the dimensions of paper if the area of the printed space is maximum?
- Q.34. Find the area cut off from the parabola  $4y = 3x^2$  by the line  $2y = 3x + 12$ .

(2) \*\*\*\*\*