Junior College

Preliminary Examination 2020-21

Std. 12th (Com.) Date - 17/2/2021 | Sub : Mathematics & Statistics | Marks - 80 Time - 11.00 To 2.00

Note :-

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. There are 6 questions divided into two sections.
4. Write answer of section I & II in the same answer book.
5. Use of logarithmic table is allowed. Use of calculator is not allowed.
6. For LPP graph paper is not necessary ; only rough sketch of graph is expected.

SECTION I

Q.1 A) Select & write the most appropriate answer from the given alternatives. 6

1. $\int e^{2x} \cdot x^2 \, dx =$ .....

   a) $e^{2x}\left(\frac{x^2}{2} - \frac{x}{4} + \frac{1}{4}\right) + c$ b) $e^{2x}\left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right) + c$

   c) $e^{2x}\left(\frac{x^2}{2} - x + \frac{1}{4}\right) + c$ d) $e^{2x}\left(\frac{x^2}{2} - \frac{x}{2}\right) + c$

2. Which of the following is always true ?
   a) $(p \to q) \equiv \sim q \to \sim p$
   b) $\sim (p \vee q) \equiv \sim p \vee \sim q$
   c) $\sim (p \to q) \equiv p \wedge \sim q$
   d) $\sim (p \wedge q) \equiv \sim p \wedge \sim q$

3. If $y = \log\left(\frac{e^x}{x^2}\right)$, then $\frac{dy}{dx}$ = ...

   a) $\frac{2-x}{x}$ b) $\frac{x-2}{x}$ c) $\frac{e-x}{ex}$ d) $\frac{x-e}{ex}$

4. The equation of tangent to the carve $y = x^2 + 4x + 1$ at (-1, -2) is
   a) $2x - y = 0$ b) $2x + y - 5 = 0$ c) $2x - y - 1 = 0$ d) $x + y = 1$

5. The solution of $x\frac{dy}{dx} = y \log y$ is

   a) $y = ae^x$ b) $y = be^{2x}$ c) $y = be^{-2x}$ d) $y = e^{ax}$

(P.T.O.)

6. $\int_2^3 \frac{x}{x^2-1}\,dx$ = ...

a) $\log\left(\frac{8}{3}\right)$ b) $-\log\left(\frac{8}{3}\right)$ c) $\frac{1}{2}\log\left(\frac{8}{3}\right)$ d) $\frac{-1}{2}\log\left(\frac{8}{3}\right)$

B) State whether each of the following is True or False. 3

1. Order & degree of a differential equation are always positive integer.
2. If $\int x\, f(x)\,dx = \frac{f(x)}{2}$ then $f(x) = e^{x^2}$
3. There are 24 months in year is a statement.

C) Fill in the blanks. 3

1. If $A=\begin{bmatrix} \frac{3}{2} & \frac{-5}{5} \end{bmatrix}$ then cofactor of a_{12} is .....

$m_{12} = 2$ $(-1)^{1+2} \cdot 2 = -2$

2. $\int_4^3 \frac{1}{\sqrt{x}}\,dx$ = ...
3. To find the value of $\int \frac{(1+\log x)}{x}\,dx$ the proper substitution is .....

Q.2 A) Attempt any Two of the following. 6

1. Evaluate $\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$
2. Find the inverse $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ of by the elementary row transformation.
3. Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}$

B) Attempt any Two of the following. 8

1. If the demand function is $D = \left(\frac{P+6}{P-3}\right)$ find the elasticity of demand at $p = 4$.
2. Using definite integration find the area of the circle $x^2 + y^2 = 25$.
3. The rate of growth of bacteria is proportional to the number present. If initially, there were 1000 bacteria & the number doubles in 1 hours, find the number of bacteria after 5/2 hours (Given $\sqrt{2} = 1.414$)

(P.T.O.)

Q.3 A) Attempt any Two of the following. 6

1. Determine whether following statement pattern is a tautology contradiction or contingency. $[(\sim p \wedge q) \rightarrow p] \leftrightarrow [\sim p \wedge \sim q]$

2. If $x = \frac{4t}{1+t^2}$, $y = 3\left(\frac{1-t^2}{1+t^2}\right)$ then show that $\frac{dy}{dx} = \frac{-9x}{4y}$

3. Using the algebra of statements, prove that
$(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv (p \vee \sim q)$

B) Attempt any One of the following. 4

1. Solve the differential equation.
$(x^2 - y^2)\,dx + 2xy\,dy = 0$

2. The sum of the cost of an Economics book one co-operation book & one Account book is ₹ 420. The total cost of an Economics book, 2 co-operation books & an Account book is ₹ 480. Also the total cost of an Economics book, 3 co-operation books & 2 Account books is ₹ 600 Find the cost of each book.

C) Attempt any One of the following. 4

1. Evaluate $\int \frac{1+\log x}{x(3+\log x)(2+3\log x)}\,dx$

Solution :

Put $t = \square$ $\therefore$ $dt = \square\,dx$

$\therefore \int \frac{1+\square}{(3+\square)(2+3\square)}\,dt$

$\therefore \frac{1+\square}{(3+\square)(2+3\square)} = \frac{A}{3+\square} + \frac{B}{2+3\square}$ by using partial fraction method

$\therefore 1+\square = A[2+3\square] + B[3+\square]$

Put $t = -3$ $\therefore$ $A = \square$

& $t = -\frac{2}{3}$ $\therefore$ $B = \square$

We got $\int \frac{\square}{3+\square}\,dt + \int \frac{\square}{2+3\square}\,dt$

$= \square \log|3+\square| + \square \log|2+3\square| + c$

2. The reactangle has area of $50cm^2$. Find its dimensions for least perimeter.

Solution :

(P.T.O.)

let length of reactangle = x cm
bredth of reactangle = y cm

$\therefore$ Area $= xy = 50$

$\therefore y = \frac{50}{\square}$

Perimeter of reactangle $= 2(x+y)$

$= 2\left[x + \frac{50}{\square}\right]$

let $f(x) = 2\left[x + \frac{50}{\square}\right]$

$f'(x) = 2\left[1 - \frac{50}{\square}\right]$

& $f''(x) = 2[0 + \square] = \frac{200}{\square}$

Consider $f'(x) = 0 \therefore 1 - \frac{50}{\square} = 0$

ie if $x^2 = \square \therefore x = \pm\square$

but x is not negative $\therefore x = \square$

& $f''\square = \frac{200}{\square} > 0$

by second derivative test, f is minimum at

$x = \square$

when $x = \square$, $y = \frac{50}{\square} = \square$

$\therefore x = \square$ cm, $y = \square$ cm

Q.4 A) Select & write the most appropriate answer from the given alternatives. 6

1. If $x \sim B(20, {}^1/_{20})$ then $F(x) =$ .....
   a) 2 b) 1 c) 4 d) 5

2. If x is r.v. with p.d.f. $f(x) = \frac{K}{\sqrt{x}}$, $0 < x < 4$
   $= 0$, otherwise, $E(x) =$ .....
   a) ${}^1/_3$ b) ${}^4/_3$ c) ${}^2/_3$ d) 1

3. You get payments of ₹ 8000 at the beginning of each for five years at 6% what is the value of this annuity ?
   a) ₹ 34,720 b) ₹ 39,320 c) ₹ 35,720 d) ₹ 40,000

(P.T.O.)

4. $|bxy + byx| \geq$ .....
   a) $|r|$ b) $2|r|$ c) r d) $2r$

5. The cost of living index number using weighted relative method is given by

   a) $\frac{\Sigma IW}{\Sigma W}$ b) $\Sigma\frac{W}{IW}$ c) $\frac{\Sigma W}{\Sigma IW}$ d) $\Sigma\frac{IW}{W}$

6. Using Hungarian method the optimal assignment obtained for the given problem to minimize the total cost is

	Jobs			
Agents	A	B	C	D
1	10	12	15	25
2	14	11	19	32
3	18	21	23	29
4	15	20	26	28

   a) 1→ C, 2 → B , 3 → D, 4 → A
   b) 1 → B, 2 → C, 3 → A, 4 → D
   c) 1 → A, 2 → B, 3 → C, 4 → D
   d) 1→ D, 2 → A, 3 → B, 4 → C

B) State whether each of the following is True / False. 3

1. Trade discount is allowed on catalogue price.

2. $\frac{\Sigma P_0\sqrt{q_0q_1}}{\Sigma P_1\sqrt{q_0q_1}} \times 100$ is Walsh's Price Index Number.

3. The p.m.f. of a r.v. x is

   $P(X = x) = \frac{2x}{n(n+1)}$, $x = 1,2$ .....n

   $= 0$, otherwise

   then $E(x) = \frac{2n+1}{3}$

C) Fill in the blanks. 3

1. The time for which a machine j does not have a job to process to the start of job λ is called .....
2. Value index Number by Simple Aggregate Method is given by .....
3. The difference between the banker's discount & the true discount is called .....

Q.5 A) Attempt any Two of the following. 6

1. Find the sequence that minimizes the total elapsed time required to complete the following tasks. The table below gives the processing time in hours. Also, find the minimum elapsed time & idle times for both the machines. (P.T.O.)

Jobs	1	2	3	4	5
M1	3	7	4	5	7
M2	6	2	7	3	4

2. A person plants to put ₹ 400 at the beginning of each year for 2 years in a deposite that gives interest at 2% p.a. compounded annually. Find the amount that will be accumulated at the end of 2 years ?
3. The following table shows the index of industrial production for the period from 1976 to 1985, using the year 1976 as the base year.

Year	1976	1977	1978	1979	1980
Index	0	2	3	3	2
Year	1981	1982	1983	1984	1985
Index	4	5	6	7	10

Obtain the trend values for the above data using 4 yearly centered moving averages.

B) Attempt any Two of the following. 8

1. A bill of ₹ 51,000 was drawn on 18th Feb. 2010 for 9 months. It was encashed on 28th June 2010 at 5% p.a. Calculate the banker's gain & true discount.
2. The two regression equations are $5x - 6y + 90 = 0$, $15x - 8y - 130 = 0$ Find $\bar{x}$, $\bar{y}$ & r .....
3. Solve the following unbalanced assignment problem of minimizing total time for doing all the jobs.

Jobs \ Persons	A	B	C	D
J1	3	6	2	6
J2	7	1	4	4
J3	3	8	5	8
J4	5	2	4	3
J5	5	7	6	2

Q.6 A) Attempt any Two of the following. 6

1. Solve the following LPP graphically Minimize $Z = 3x-y$ subject to constraints $2x + y \geq 2$, $x+3y \leq 2$, $y \leq 2$, $x,y \geq 0$
2. In a cattle breeding firm, it is prescribed that the food ration for one animal must contain 14,22 & 1 unit of nutrients A,B & C respectively. Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of these three nutrient. The cost of a fodder 1 is ₹ 3 per unit & that of fodder 2 is ₹ 2 per unit. Formulate the LPP to minimize the total cost.

Nutrients \ Fodder	Fodder 1	Fodder 2
A	2	1
B	2	3
C	1	1

3. For the following data, find the regression line of y on x : Hence find the most likely value of y when x = 4

X	1	2	3
Y	2	1	6

(P.T.O.)

4

B) Attempt any One of the following.

1. Find the expected value & variance of the r.v.x. if its probability distribution is as follows.

x	1	2	3	.....	n
P (X = x)	1/n	1/n	1/n	.....	1/n

2. The following table gives the production of steel (in millions of tonnes) for years 1976 to 1986. Fit a trend line to the data by the method of least squares. Also, obtain the trend value for the year 1990.

Year	1976	1977	1978	1979	1980	1981
Production	0	4	4	2	6	8
Year	1982	1983	1984	1985	1986	-
Production	5	9	4	10	10	-

4

C) Attempt any One of the following.

1. Find x, if Walsch's Price Index Number is 140 for the following data :

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	20	8	40	7
B	50	10	60	10
C	40	15	60	x
D	12	15	15	15

Solution

Commodity	Base Year		Current Year			
	p_0	q_0	p_1	q_1	p_1q_1	p_0q_1
A	20	8	40	7	280	140
B	50	10	60	10	600	500
C	40	15	60	x	☐	☐
D	12	15	15	15	225	180
				Total	$\Sigma p_1q_1 =$ ☐	$\Sigma p_0q_1 =$ ☐

Poasch's Price index Number

$$Po1(P) = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100$$

$$= \frac{\square}{\square} \times 100$$

but P01 (P) = 140 ..... given

$$\therefore 140 = \frac{\square}{\square} \times 100$$

$$\therefore 7 = \frac{\square}{\square} \times 5$$

$$\therefore x = \square$$

40.8
40.8 × 2 / 100
4.08 4.16 × 2 ×
× 2
8.16

(P.T.O.)

2. If x has poisson distribution with parameter m & P(x=2) = P(x=3), then find p (x ≥ 2). Use $e^{-3} = 0.0497$.

Solution

$x \sim p(m)$

$\therefore p[X = x] = \frac{e^{-m} m^{x}}{\square}$

Given $p[x = 2] = p[x = 3]$

$\frac{e^{-m} m^{\square}}{\square} = \frac{e^{-m} m^{\square}}{\square}$

$\therefore m = \square$

$\therefore P[x \geq 2] = 1 - [p(x = \ldots\ldots) + p(x = \ldots\ldots)]$

$= 1 - (\square + \square)$

$= 1 - \square$

$= \square$

*****